AN INVESTIGATION OF SEMI-DIURNAL
FLUCTUATIONS OF WINDS AT AN
ALTITUDE OF TEN KILOMETERS

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1. Introduction.

The study of atmospheric tidal oscillations has been the subject of long and intense research by many eminent scientists. Its history and theory have been particularly well recounted by Chapman [2] and Wilkes [4]. The regular 24-hourly fluctuations of pressure have been observed and analyzed since the invention of the barometer. More recently the air motions of the ionosphere have received special attention. Although solar diurnal variations of the winds in the troposphere and stratosphere have been the subject of inference from somewhat over-simplified theories, observations of such effects have been meager.

The purpose of this study is to make an analysis of a possible solar semi-diurnal variation in the winds at ten kilometers from the regular upper wind observations published in the Northern Hemisphere Data Tabulations [5].

2.   Data selection.

The ten kilometer winds during a 100-day period from
1 November 1959 through 8 February 1960 observed at 15 stations
were extracted from the Northern Hemisphere Data Tabulations [5].
Nine of these stations recorded the ten kilometer wind at 0000Z,
0600Z, 1200Z, and 1800Z with reasonable regularity each day. The
other six stations observed the ten kilometer wind at 0000Z and
1200Z only. As a maximum of only four observations per day
presents considerable impediment to satisfactory analysis of a
semi-diurnal oscillation, the stations were selected in a spatial
relationship designed to indicate the wave length of the oscilla-
tion. Also a period was chosen to provide relative constancy of
certain solar tidal parameters. The period selected is symmetrical
to the northern hemisphere winter - the sun's declination pro-
gressing from about 15 degrees south on 1 November through the
solstice on 22 December and back to about 15 degrees south on
8 February. The location of the stations was such that they
extended over approximately 180 degrees of longitude with the
goal of encompassing spatially one complete semi-diurnal wave.
As there is empirical evidence [3] for a meridionally-directed
standing pressure wave of solar semi-diurnal character with a
node at 35 degrees latitude, the stations were selected primarily
in the latitude belt 30 to 35 degrees north in order to minimize
the wind effects of the standing wave.

3. Data analysis.

The westerly component of each six-hourly wind observation was computed and tabulated by station, date, and time. For each six-hourly time and at each station the average was taken. These results are tabulated in Table 1 along with the total number of observations comprising each average.

Data from each of the nine stations recording four wind observations per day was treated first by the method of differences between each pair of successive six-hourly averages. Assuming then that the change in westerly component over a 12-hour period is characteristic of a 24-hour wave and that one-half of this change occurs in each of the included six-hour periods, the diurnal effect is approximately eliminated by subtracting one-half of the 12 hour change from each of the six-hourly changes. This process is applied over the periods 0000Z to 1200Z and from 1200Z to 0000Z. This method of analysis gives an apparent six-hourly change at each station of the form:

$$00Z \text{ to } 06Z : \Delta_1 u = x \qquad ; \qquad 12Z \text{ to } 18Z : \Delta_3 u = y$$

$$06Z \text{ to } 12Z : \Delta_2 u = -x \qquad ; \qquad 18Z \text{ to } 00Z : \Delta_4 u = -y$$

Since x and y are in general unequal, the differences may be accounted for by a terdiurnal wave. For simplicity let this wave be symbolized by T, and assume it to be of constant amplitude and phase at each station during the observation period. Then the four differences just formed, $\pm x$ and $\pm y$, include terdiurnal differences $T_{06} - T_{00}$, $T_{12} - T_{06}$, $T_{18} - T_{12}$, and $T_{00} - T_{18}$. Hence $\Delta_1 u$, $\Delta_2 u$, $\Delta_3 u$, and $\Delta_4 u$ may be written in

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the form:

$$\Delta_1 u = x' + (T_{06} - T_{00}) \qquad ; \qquad \Delta_3 u = x' + (T_{18} - T_{12})$$

$$\Delta_2 u = -\left[x' - (T_{12} - T_{06})\right] \quad ; \quad \Delta_4 u = -\left[x' - (T_{00} - T_{18})\right]$$

where x' is the desired six-hour difference based on a <u>semi-diurnal wave alone</u>. Then may be formed

$$\Delta_1 u - \Delta_2 u + \Delta_3 u - \Delta_4 u = 4x' + 2(T_{06} + T_{18}) - 2(T_{00} + T_{12}) \quad (1)$$

Since points on the terdiurnal wave which are 12 hours apart are 180 degrees out of phase, it follows that $T_{00} + T_{12} = 0$ and $T_{06} + T_{18} = 0$. Hence equation (1) eliminates the terdiurnal wave. Since the 24 hour wave had been previously eliminated, equation (1) gives four times the "amplitude" of the semi-diurnal change for the particular phase of the wave at the given station. In other words, the mean magnitude of the six-hour difference at any station is

$$\frac{\Delta_1 u - \Delta_2 u + \Delta_3 u - \Delta_4 u}{4} = x' \qquad\qquad (2)$$

and the four differences freed of both 24-hour and eight-hour harmonics are

    00Z to 06Z : x'   ;   12Z to 18Z : x'

    06Z to 12Z : -x'   ;   18Z to 00Z : -x'

The four differences shown above will be termed the "filtered" semi-diurnal six-hour differences.

Oscillations of shorter periods which are evenly divisible into six have been removed also by the technique just described.

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A four-hour period wave may not have been removed by this process; however, a four-hour wave of significant amplitude is unknown at the present time.

As the 100-day period selected is of such duration to comprise three and one-half oscillations through the moon's range of declination and phase, lunar tidal effects are deemed to be small. For more nearly complete elimination of the lunar effect, selection of a period comprising at least 12 complete lunar oscillations (approximately 336 days) would have been more appropriate; however, the lunar tidal oscillation is generally recognized to be small compared to the solar semi-diurnal wave.

Appendix 1 contains the filtering calculations for each of the nine stations which report four times daily. Figures 1 through 9 depict graphically these filtered six-hour changes and the corresponding observed six-hourly means of the westerly wind component at each station. The filtered six-hour changes have been plotted in such a way as to divide the quantities x', $-x'$ symmetrically about the 100-day mean. Thus if a positive change occurs between 0000Z and 0600Z, a value $x'/2$ above the mean was plotted at 0600Z while at 0000Z a value $-x'/2$ was plotted. Examination of Figures 1 through 9 shows that the observed mean six-hour changes frequently have two maxima and two minima in the 24-hour period and are in this respect and in essential form similar to the filtered six-hour differences unless the latter are so small as to be obscured by "noise".

A further analysis has been performed by plotting the

filtered fluctuations over the full range of longitude of all
nine stations. Figures 10 and 11 depict these spatially-arranged
fluctuations for the periods 0000Z to 0600Z and 0600Z to 1200Z
respectively. For contrast, Figure 12 was prepared presenting
the observed 12-hour changes for all 15 stations also plotted
over the full range of longitude. The curves inserted in these
figures have been fitted by eye, and in all cases the vertical
scale is the same to help convey relative magnitudes.

Station	No.	Lat.	Long.	0000Z	N	0600Z	N	1200Z	N	1800Z	N	Daily Mean	N
Kenitra	119	34.3N	6.6W	15.79	96			15.72	92			15.76	188
Ship	034	35.0N	48.0W	20.53	72	18.18	40	18.69	68	14.66	38	18.50	218
Norfolk	308	36.9N	76.2W	37.71	90	37.26	90	37.05	91	38.61	96	37.78	367
Charleston	208	32.9N	80.0W	31.30	83			31.91	81			31.60	164
Montgomery	226	32.3N	86.4W	33.77	93	35.17	93	33.04	93	33.66	97	33.91	376
Shreveport	248	32.5N	93.8W	31.32	99	30.30	99	31.41	99	30.51	96	30.89	393
Fort Worth	259	32.8N	97.1W	31.15	97	31.44	96	31.36	96	29.72	95	30.92	384
Tuscon	247	32.1N	110.9W	24.81	91			24.60	91			24.71	182
San Diego	290	32.7N	117.2W	17.56	96			20.61	92			19.05	188
Santa Monica	295	33.9N	118.4W	15.97	95	17.31	96	18.73	90	16.66	96	17.16	377
Ship	134	30.0N	140.0W	14.41	80	10.94	48	15.40	81	8.03	38	13.08	247
Hilo	285	19.7N	155.1W	17.19	91			17.62	86			17.42	177
Midway	066	28.2N	177.4W	22.27	91	24.78	83	24.32	95	25.78	86	24.26	355
Wake	245	19.3N	166.6E	3.44	99			3.71	100			3.58	199
Ship	236	34.0N	164.0E	53.80	66	40.27	22	51.83	71	45.57	28	50.23	187

Table 1. Average westerly component of the wind at ten kilometers in meters per second. (N=Number of observations comprising the average preceeding)
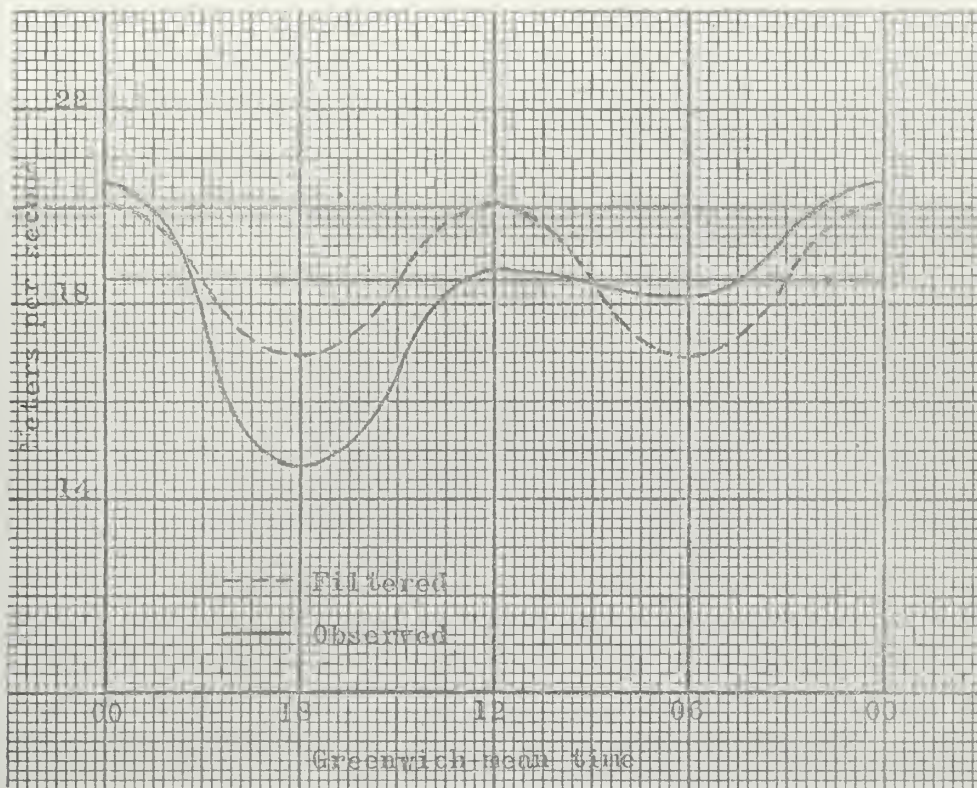
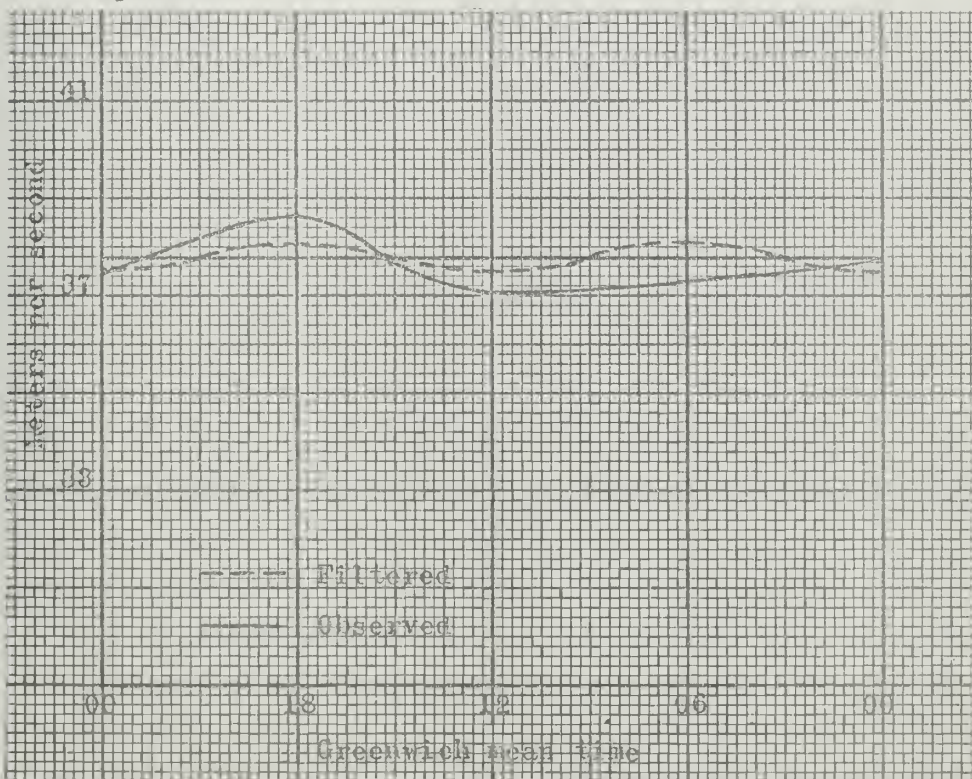Fig. 1. Average westerly component of the wind at ten kilometers at Ship 034.

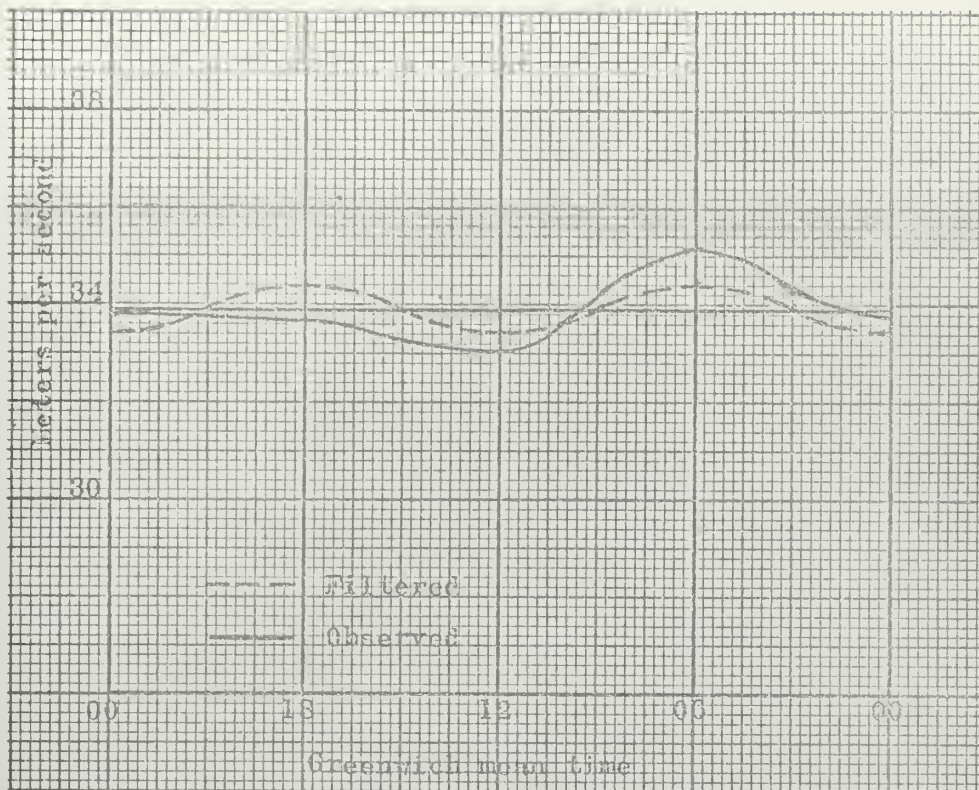Fig. 2. Average westerly component of the wind at ten kilometers at Norfolk

Fig. 3. Average westerly component of the wind at ten kilometers
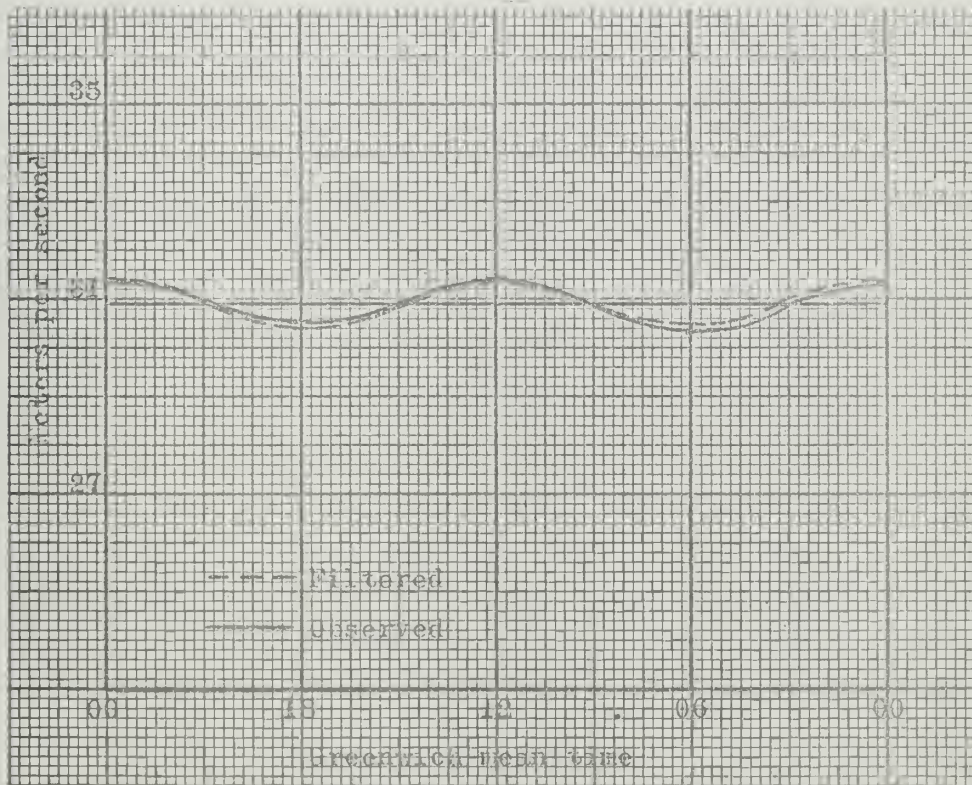at Montgomery.

Fig. 4. Average westerly component of the wind at ten kilometers
at Shreveport.

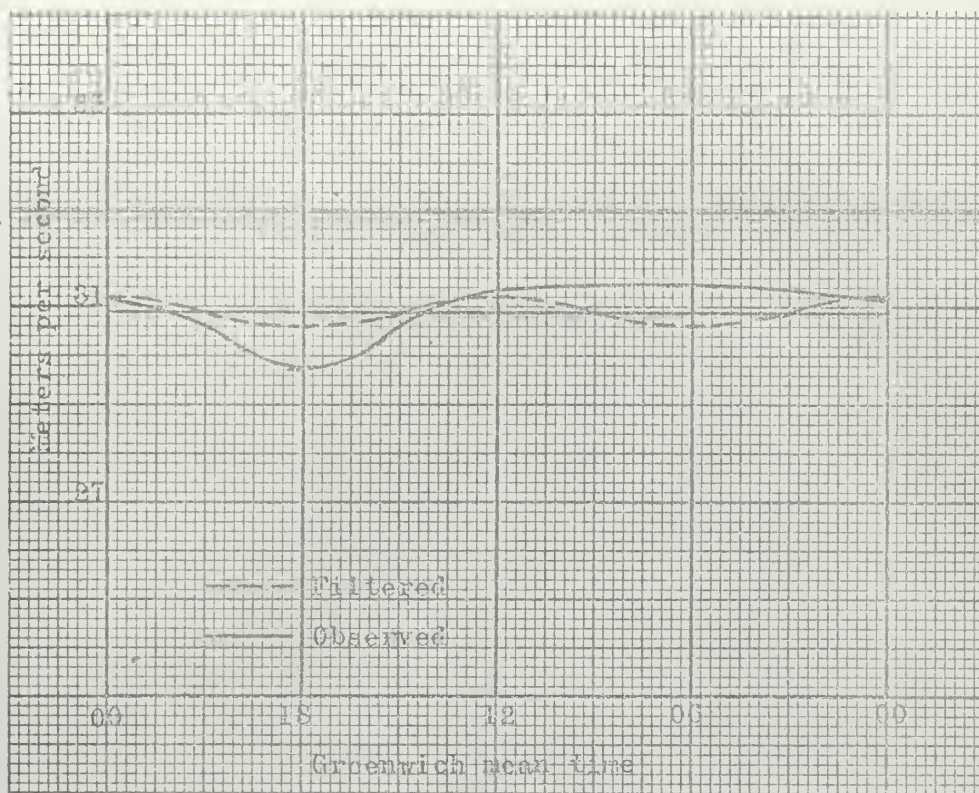Fig. 5. Average westerly component of the wind at ten kilometers at Fort Worth.

Fig. 6. Average westerly component of the wind at ten kilometers at Santa Monica.
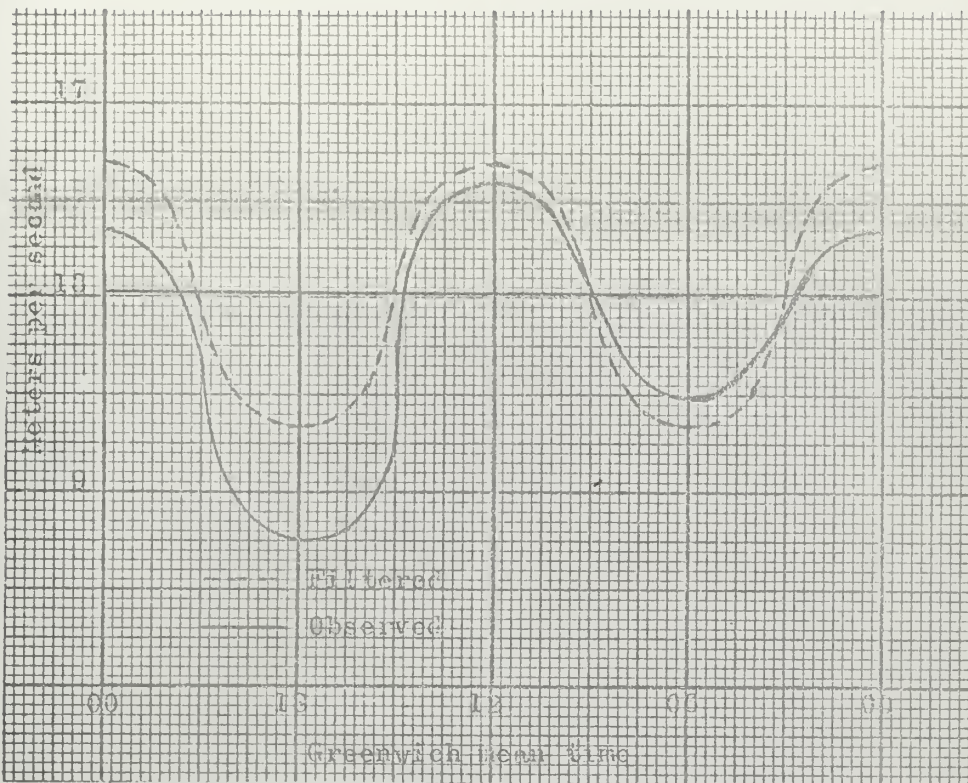
Fig. 7. Average westerly component of the wind at ten kilometers at Ship 134.

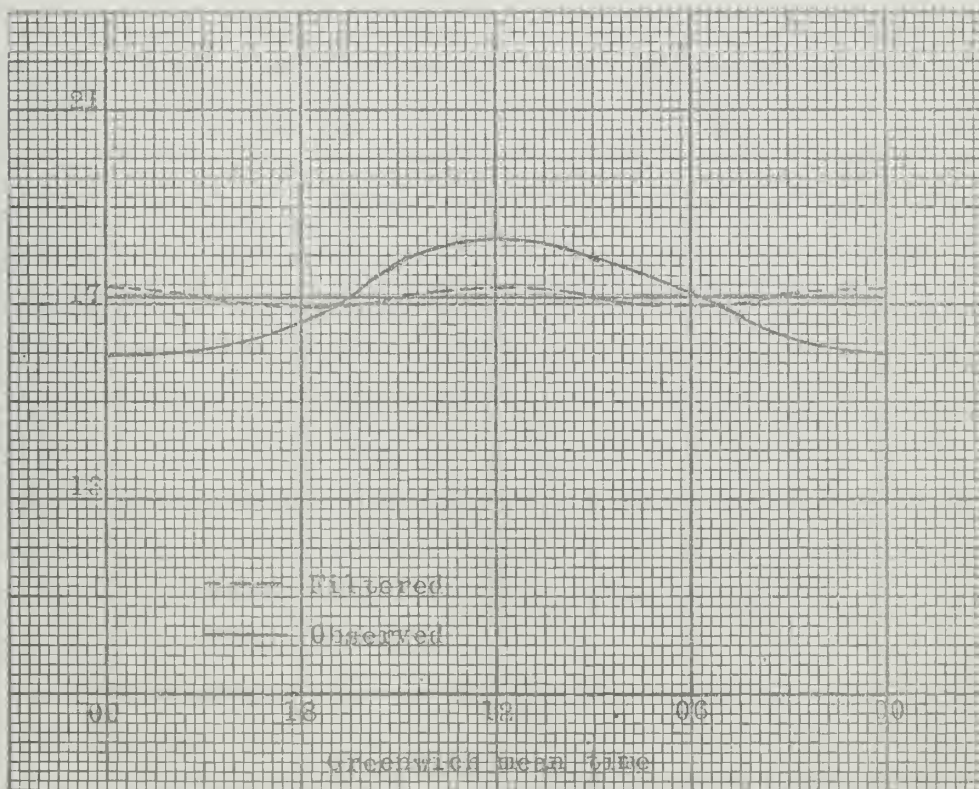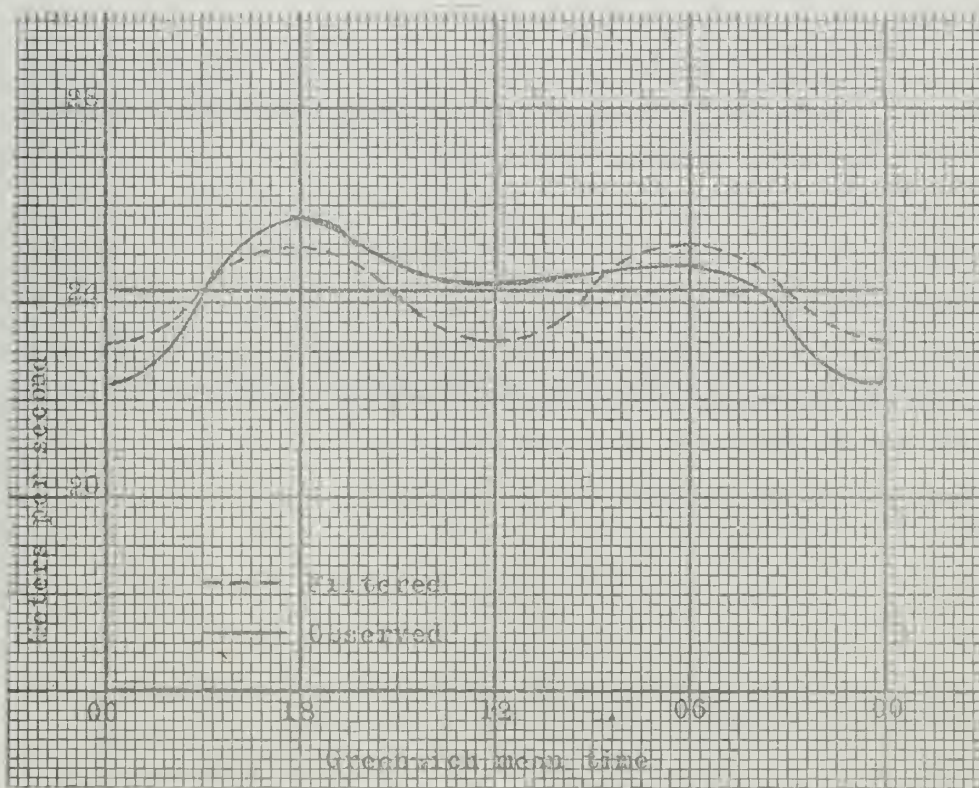Fig. 8. Average westerly component of the wind at ten kilometers at Midway.
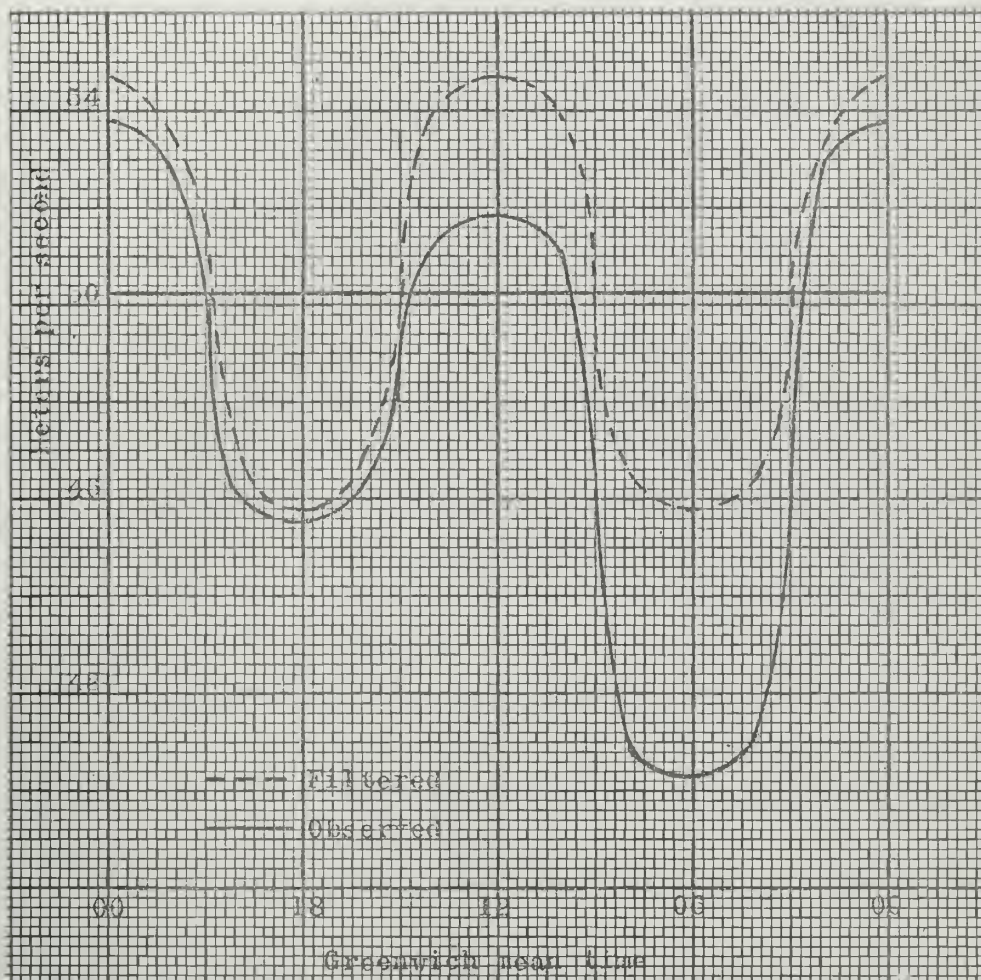
Fig. 9. Average westerly component of the wind at ten kilometers at Ship 236.
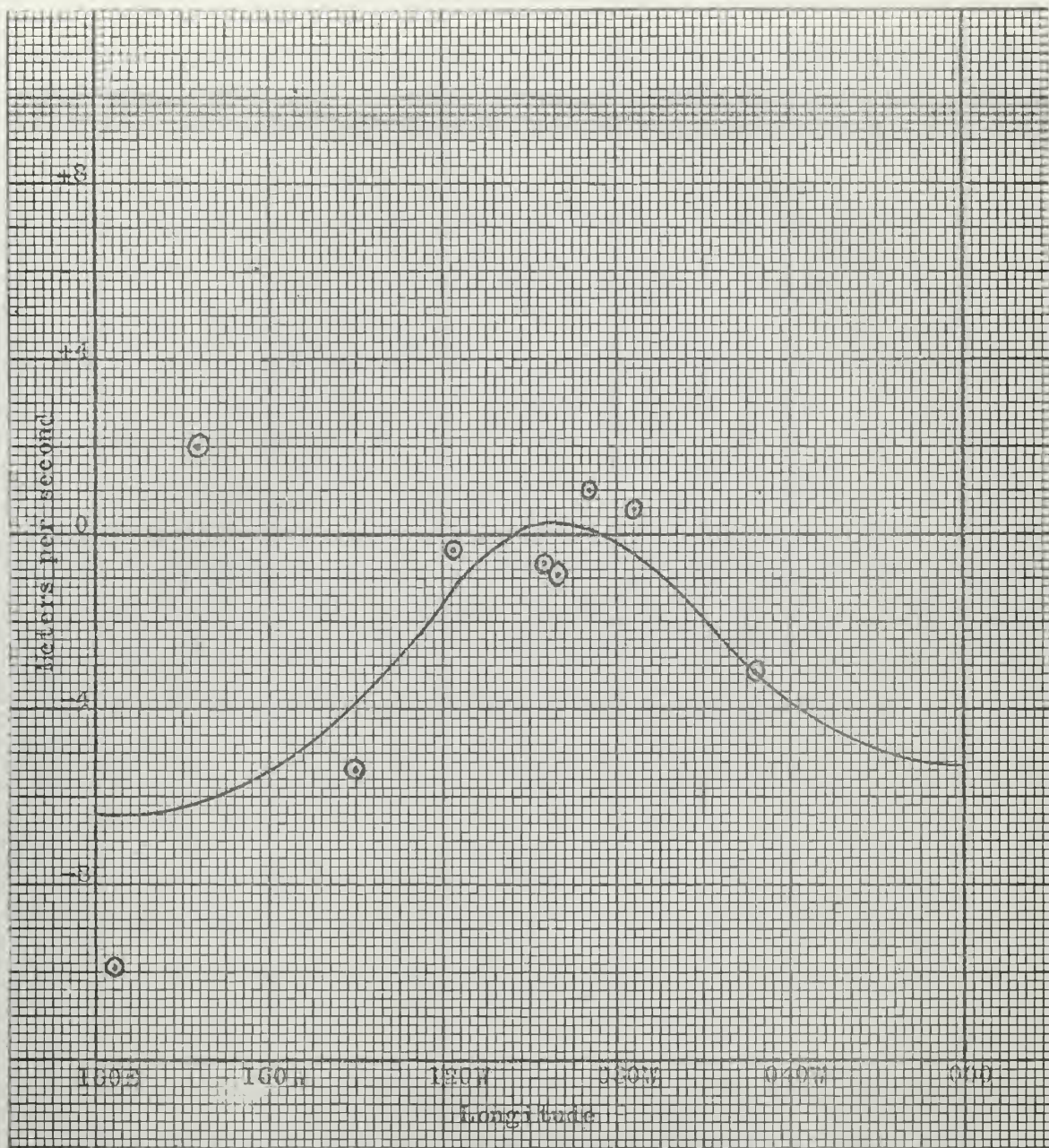
Fig. 10. Filtered six-hour variations in the average westerly component of the wind at ten kilometers for the periods 0000Z to 0600Z and 1200Z to 1800Z.
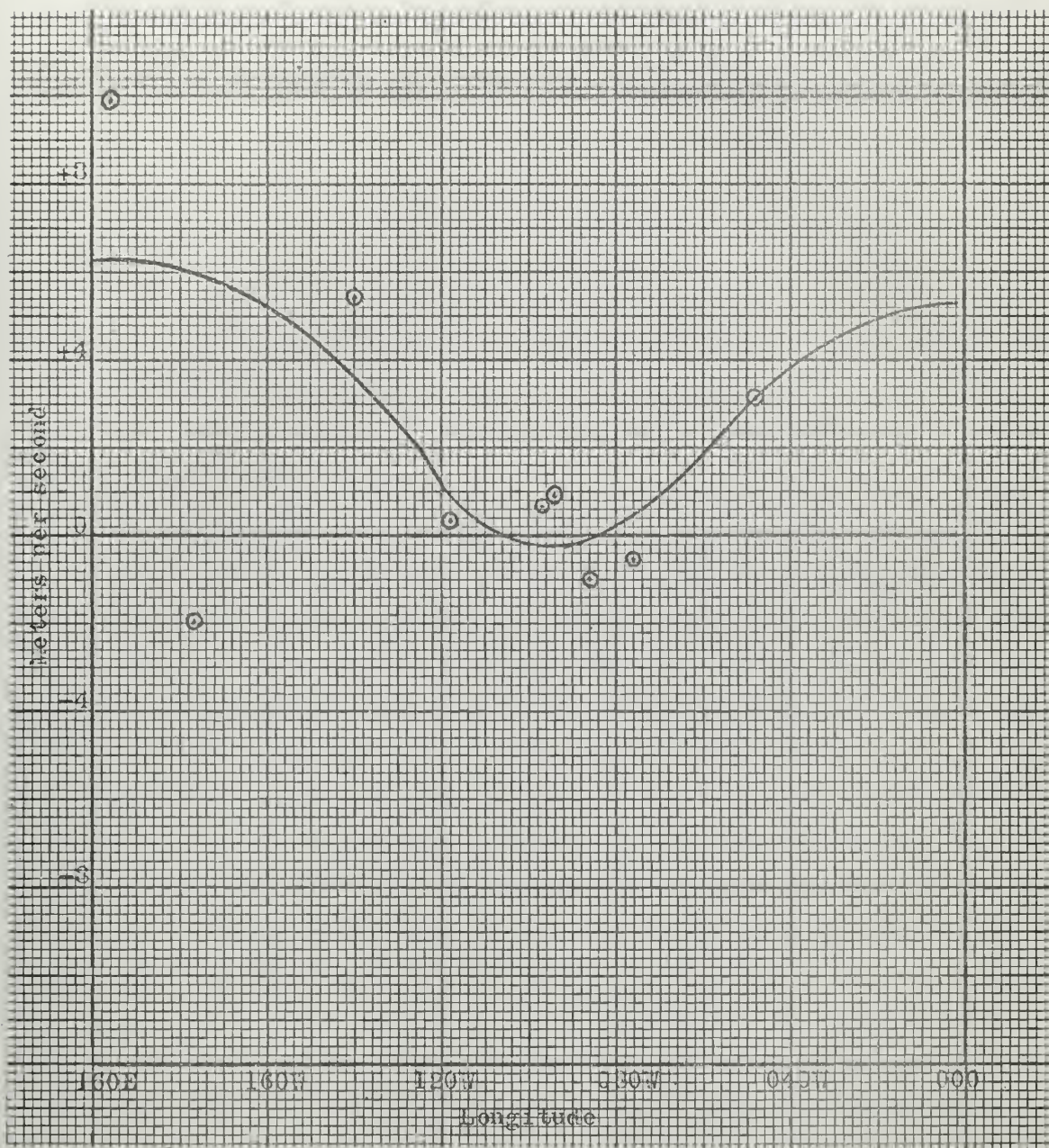
Fig. 11. Filtered six-hour variations in the average westerly
component of the wind at ten kilometers for the periods 0600Z
to 1200Z and 1800Z to 0000Z. This curve is a mirror-image of
that in Fig. 10.

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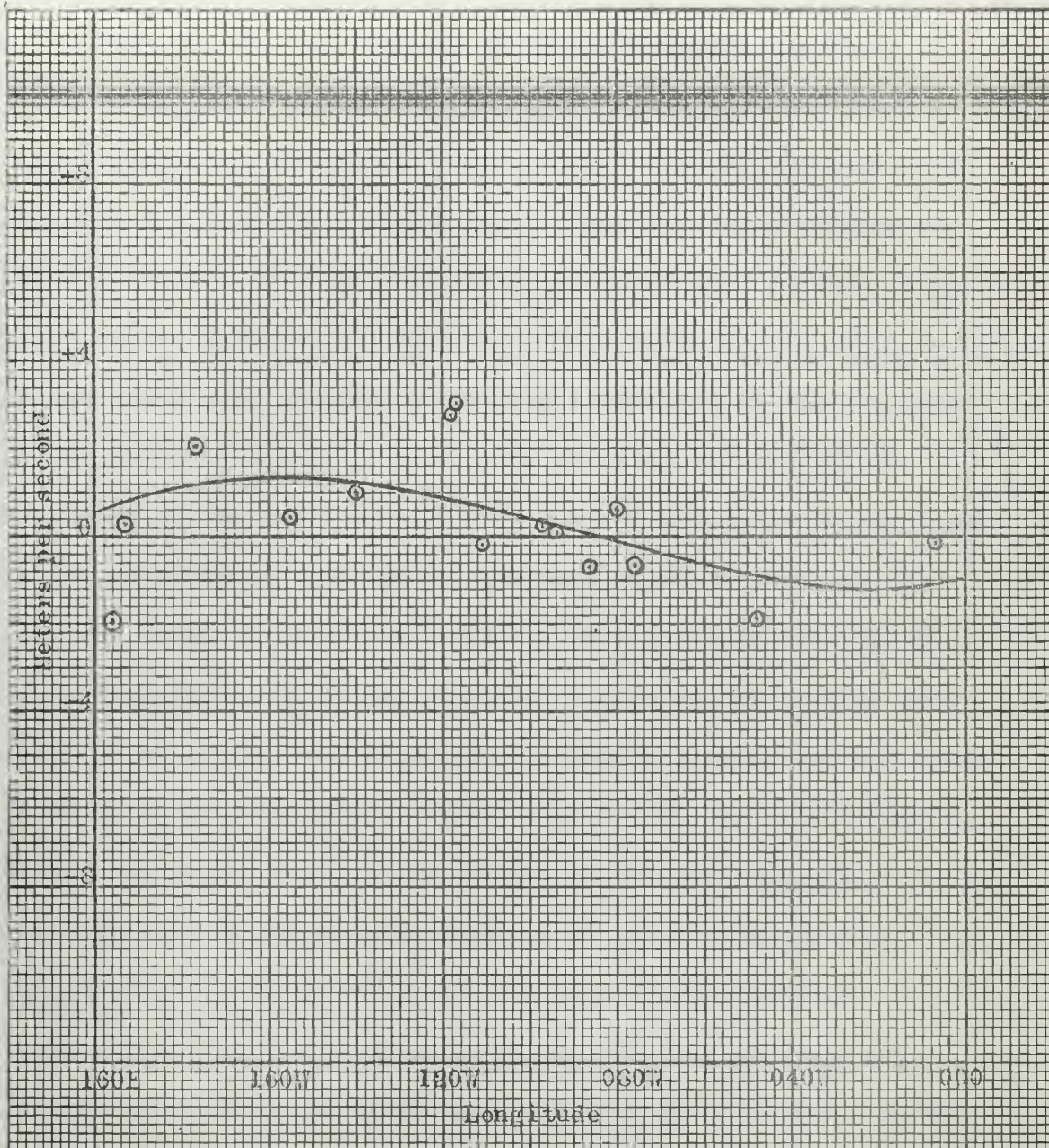
Fig. 12. Observed 12-hour variation in the average westerly component of the wind at ten kilometers for the period 0000Z to 1200Z.

4.   Discussion of results.

If the mean values of an element are available at each solar hour of the day, derivation through harmonic analysis of the 24-hour wave, as well as the semi-diurnal wave and some higher frequency harmonics, is relatively simple. This has been done for the surface pressure wave (see, for example, Chapman [2] and Wilkes [4]) and for other elements. As a further example, Greenhow and Neufeld [6] have isolated a semi-diurnal oscillation of the zonal wind between 80 and 100 kilometers by observations of the drift of meteor trails. More recently Mantis [9] has investigated the existence of both diurnal and semi-diurnal waves in the meridional component of the wind at 29 kilometers. His data was based on drifts of constant pressure balloons. On the other hand Pekeris [8] has presented a more sophisticated theory in which he assumes a temperature structure similar to that known to occur in the atmosphere. Using essentially the Pekeris model, Wilkes has shown that a phase change of 180 degrees occurs in the pressure wave in the neighborhood of 30 kilometers; however, at ionospheric levels the phase of the semi-diurnal pressure wave is very nearly again in phase with the observed surface semi-diurnal wave, as is approximately required by the dynamo theory of magnetic variations. According to the results obtained in this paper, the change in phase (by 180 degrees) appears to have occurred as low as ten kilometers, and the semi-diurnal wave in the zonal component has an amplitude of two meters per second at this same level over oceanic stations.

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In the case of most upper-air meteorological data, however, one can expect no more than four synoptic reports per day. Hence the usual harmonic analysis procedures cannot be performed. For this reason, the technique of the filtered six-hour changes was devised utilizing the requirement of additional verification by proper spatial-temporal phase relationship to support the postulate of a 12-hour wave at ten kilometers.

Assuming a semi-diurnal wave at each station, the times of maxima or minima can be inferred from the filtered six-hour changes only to within ± 3 hours of the filtered maxima and minima. The best that can be said of the amplitude from a single station record is that it should be not less than half the filtered change. Although only a few of the nine stations appear to have significant filtered six-hour changes, the spatial configuration of these wind variations depicted in Figures 10 and 11 lends credence to their semi-diurnal character and extent. There appears to be a fairly good indication that the adjacent maxima and minima are very close to 90 degrees of longitude or six hours apart. Except for the observations at Midway, the amplitudes and phases over the oceans suggest a sinusoidal character. A possible explanation for the much smaller amplitude over the United States may be the presence of more intense vertical motions associated with persistent long wave troughs frequent in winter there, which could obscure the horizontal fluctuations sought. The results obtained at Midway in opposition to other indications may be due to its departure in latitude by

five degrees from the mean latitude of all the stations. This small latitude difference, however, does not strongly support such an explanation.

The phase of the wind fluctuation depicted in Figures 10 and 11 deserves some comment because it is opposite to that described in the well-known text by Mitra [3] . He gives a simplified theory using an atmosphere of constant density. According to this theory the surface semi-diurnal wind is in phase with the semi-diurnal pressure wave, having a maximum easterly component at approximately ten a.m. and ten p.m. local time. This would mean that the prevailing westerlies at 33 degrees north latitude should be diminished at these local times. Figures 10 and 11, however, show the westerly wind to be greater than the mean during the period 0600 to 1200 local time.

A statistical verification of certain of these results was considered in the following manner: Assuming a solar diurnal wave of 24-hour period of similar form as the assumed terdiurnal wave and symbolizing this wave with "S", that portion of the westerly component of the wind due to tidal effects may be con-sidered the sum of x_i, the semi-diurnal wave, T, the terdiurnal wave, and S. This may be stated as follows:

$$u_{00Z} = x_{i,00Z} + T_{00Z} + S_{00Z} + \bar{u} \text{ , and}$$

$$u_{12Z} = x_{i,12Z} + T_{12Z} + S_{12Z} + \bar{u} \text{ .}$$

where $\bar{u}$ is the mean zonal wind for the period of study. Adding yields

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$$u_{00Z} + u_{12Z} = x_{i,00Z} + x_{i,12Z} + T_{00Z} + T_{12Z} + S_{00Z} + S_{12Z} + 2\bar{u} \, ,$$

but $T_{00Z} + T_{12Z} = 0$ as before, and $S_{00Z} + S_{12Z} = 0$ also. The mean of the semi-diurnal components 12 hours apart thus corresponds to the mean of the zonal winds taken 12 hours apart. Thus

$$\tfrac{1}{2}(x_{i,00Z} + x_{i,12Z}) = \tfrac{1}{2}(u_{00Z} + u_{12Z}) - \bar{u} \, ,$$

and similarly

$$\tfrac{1}{2}(x_{i,06Z} + x_{i,18Z}) = \tfrac{1}{2}(u_{06Z} + u_{18Z}) - \bar{u} \, .$$

In other words the daily mean of the 0000Z and 1200Z zonal wind observations may be considered to be a point on the semi-diurnal wave one-half period out of phase with the mean of the 0600Z and 1800Z observations. The assumption must be made, however, that the number of 0000Z and 1200Z observations are equal (or nearly so), and that the same is true for the number of 0600Z and 1800Z reports. As the greatest semi-diurnal variation in the westerly component of the wind appeared to exist over the ocean stations, the Student's t test was applied at Ships 034, 134, and 236 to test the hypothesis that the means of the 0000Z and 1200Z observations were significantly different from the means of the 0600Z and 1800Z observations as the filtering analysis had indicated. Table 1 shows that the number of observations at each of these stations reasonably fulfills the required condition of equality.

Utilizing the symbolism and method of Hoel [7] where

x = the 0000Z and 1200Z observations,

y = the 0600Z and 1800Z observations,

n = the number of observations,

$$s^2 = \frac{\sum x_i^2}{n} - \bar{x}^2 = \text{the square of the variance, and}$$

$$v = n_x + n_y - 2 = \text{degrees of freedom,}$$

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{n_x s_x^2 + n_y s_y^2}} \sqrt{\frac{n_x n_y (n_x + n_y - 2)}{n_x + n_y}} \;.$$

Computation of t yields the following results with corresponding probability that the means of x and y above are significantly different:

Ship 034 : t = 1.15 : 87% ,

Ship 134 : t = 2.16 : 98% , and

Ship 236 : t = 3.34 : 99% .

These results lead to the conclusion that the apparently significant difference between wind observations at 0000Z and 1200Z and those at 0600Z and 1800Z is real and may be due to a semi-diurnal effect.

5.   Summary and conclusions.

Previous studies calculate an expected semi-diurnal variation of winds at the surface of the order of one-half meter per second. There appears from this investigation suggestions of much greater semi-diurnal variations in the winds at the ordinary synoptic levels, particularly over the ocean areas. Such magnitudes may require semi-diurnal modification of vertical shear prognostications so important in rocketry. This and other possible applications may be reason for more extensive study of an apparently neglected meteorological parameter.

Though accurate harmonic analysis of the various tidal effects requires more or less continous observation, consideration of the simultaneous combination of both spatial and temporal features of synoptically observed parameters may yield significant results.

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FILTERING CALCULATIONS

The calculations involved in the analysis of the observed six-hour changes in the westerly wind component are completed as follows:

Let u = average westerly component of the wind.

$$u_{06Z} - u_{00Z} = \Delta_1 u \quad ; \quad u_{18Z} - u_{12Z} = \Delta_3 u \quad ;$$

$$u_{12Z} - u_{06Z} = \Delta_2 u \quad ; \quad u_{00Z} - u_{18Z} = \Delta_4 u \quad .$$

Elimination of the 24-hour wave is accomplished as follows:

$$\Delta_1 u - \left(\frac{\Delta_1 u + \Delta_2 u}{2}\right) = x \quad ; \quad \Delta_3 u - \left(\frac{\Delta_3 u + \Delta_4 u}{2}\right) = y \quad ;$$

$$\Delta_2 u - \left(\frac{\Delta_1 u + \Delta_2 u}{2}\right) = -x ; \quad \Delta_4 u - \left(\frac{\Delta_3 u + \Delta_4 u}{2}\right) = -y \quad .$$

Elimination of the eight-hour wave is accomplished as follows:

$$\frac{\Delta_1 u - \Delta_2 u + \Delta_3 u - \Delta_4 u}{4} = x' \quad .$$

This computation resulting in the filtered semi-diurnal wave may be plotted in station model form as follows:

$$\frac{u_{00Z}}{u_{06Z}} \quad \frac{u_{12Z}}{u_{18Z}}$$

$$\frac{\Delta_1 u}{\Delta_2 u} \quad \frac{\Delta_3 u}{\Delta_4 u}$$

$$\frac{\Delta_1 u + \Delta_2 u}{2} \quad \frac{\Delta_3 u + \Delta_4 u}{2}$$

$$\frac{x}{-x} \quad \frac{y}{-y} \quad .$$

$$\frac{x'}{-x'} \quad \frac{x'}{-x'}$$

Specific calculations for each of the nine stations

recording the ten kilometer wind four times per day follow.

Ship	034	Norfolk	308	Montgomery	226
35.0N	48.0W	36.9N	76.2W	32.3N	86.4W
20.53	18.69	37.71	37.05	33.77	33.04
18.18	14.66	37.26	38.61	35.17	33.66
−2.35	−4.03	−0.45	+1.56	+1.40	+0.62
+0.51	+5.87	−0.21	−0.90	−2.13	+0.11
−0.92	+0.92	−0.33	+0.33	−0.36	+0.36
−1.43	−4.95	−0.12	+1.23	+1.76	+0.26
+1.43	+4.95	+0.12	−1.23	−1.77	−0.25
−3.14	−3.14	+0.56	+0.56	+1.01	+1.01
+3.14	+3.14	−0.56	−0.56	−1.01	−1.01

Shreveport	248	Fort Worth	259	Santa Monica	295
32.5N	93.8W	32.8N	97.1W	33.9N	118.4W
31.32	31.41	31.15	31.36	15.97	18.73
30.30	30.51	31.44	29.72	17.31	16.66
−1.02	−0.90	+0.29	−1.64	+1.34	−2.07
+1.11	+0.81	−0.08	+1.43	+1.42	−0.69
+0.05	−0.05	+0.11	−0.11	+1.38	−1.38
−1.07	−0.85	+0.18	−1.53	−0.04	−0.69
+1.06	+0.86	−0.19	+1.54	+0.04	+0.69
−0.96	−0.96	−0.63	−0.63	−0.37	−0.37
+0.96	+0.96	+0.63	+0.63	+0.37	+0.37

Ship	134	Midway	066	Ship	236
30.0N	140.0W	28.2N	177.4W	34.0N	164.0E
14.41	15.40	22.27	24.32	53.80	51.83
10.94	8.03 .	24.78	25.78	40.27	45.27
−3.47	−7.37	+2.51	+1.46	−13.53	−6.26
+4.46	+6.38	−0.46	−3.51	+11.56	+8.23
+0.50	−0.50	+1.03	−1.03	−0.99	+0.99
−3.97	−6.37	+1.48	+2.49	−12.54	−7.25
+3.96	+6.38	−1.49	−2.48	+12.55	+7.24
−5.42 ·	−5.42	+1.99	+1.99	−9.90 ·	−9.90
+5.42	+5.42	−1.99	−1.99	+9.90	+9.90

Additional stations recording the wind at 0000Z and 1200Z only are plotted in model as follows:

$$u_{00Z} \qquad u_{12Z}$$

$$u_{12Z} - u_{00Z} \qquad u_{00Z} - u_{12Z}$$

Kenitra	119	Charleston	208	Tuscon	274
34.3N	6.6W	32.9N	80.0W	32.1N	110.9W
15.79	15.72	31.30	31.91	24.81	24.60
−0.07	+0.07	+0.61	−0.61	−0.21	+0.21

San Diego	290	Hilo	285	Wake	245
32.7N	117.2W	19.7N	155.1W	19.3N	166.6E
17.56	20.61	17.19	17.62	3.44	3.71
+3.05	−3.05	+0.43	−0.43	+0.27	−0.27